

Axiomatic Language

A Short Presentation

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Intl. Conf. On Logic Programming
Las Cruces, New Mexico
September, 2019

<http://axiomaticleanguage.org/>

Language Goals

1. pure specification – what, not how
2. minimal, but extensible
3. metalanguage – can subsume other languages

Specification by Enumeration

Idea: Program external behavior defined by infinite set of symbolic expressions that enumerate inputs and corresponding outputs.

Recipe

- pure, definite Prolog with Lisp syntax
- higher-order generalization [HiLog]
- string variables

The Core Language (1)

Axioms generate valid expressions.

expression:

atom, — `abc, `+

expression variable, — %x, %1

sequence of ≥ 0 expressions and **string variables**

— (`M () \$1 %)

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axiom: conclusion expr and ≥ 0 condition exprs

<conclu> < *<cond1>*, ..., *<condn>*.

<conclu>. ! unconditional axiom

The Core Language (2)

axiom instance: substitute values for variables in axiom

$$\begin{aligned} & (\text{'A } \%x \text{ } \$1) < (\text{'B } \%x), \text{ } (\text{'C } \$1). \\ \rightarrow & \text{ (}'A \text{ } `x \text{ } `u \text{ } ()) < (\text{'B } `x), \text{ } (\text{'C } `u \text{ } ()) . \end{aligned}$$

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valid expressions: If all conditions of an axiom instance are valid expressions, the conclusion is valid.

$$\begin{aligned} & (\text{'a } `b). \\ & ((\%) \ $ \$) < (\%, \$). \\ \rightarrow & (\text{'a } `b), \\ & (((`a) \ `b \ `b), \\ & ((((`a)) \ `b \ `b \ `b \ `b), \\ & \dots \end{aligned}$$

Example – Natural Numbers

Set of natural numbers:

$$\begin{aligned} & (\text{'num } (\text{'z})). \\ & (\text{'num } (\text{'s } \$)) < (\text{'num } (\$)). \\ & \rightarrow (\text{'num } (\text{'s } \text{'s } \text{'z})) \end{aligned}$$

Addition of natural numbers:

$$\begin{aligned} & (\text{'plus } \%n \text{ } (\text{'z}) \text{ } \%n) < (\text{'num } \%n). \\ & (\text{'plus } \%1 \text{ } (\text{'s } \$2) \text{ } (\text{'s } \$3)) < \\ & \quad (\text{'plus } \%1 \text{ } (\$2) \text{ } (\$3)). \\ & \rightarrow (\text{'plus } (\text{'s } \text{'z}) \text{ } (\text{'s } \text{'z}) \text{ } (\text{'s } \text{'s } \text{'z})) \end{aligned}$$

Syntax Extensions

single char in single quotes:

```
'A' = (`char (^0 `1 `0 `0 `0 `0 `0 `1))
```

char string in single quotes within sequence:

```
(... 'abc' ...) = (... 'a' 'b' 'c' ...)
```

char string in double quotes:

```
"abc" = ('abc') = ('a' 'b' 'c')
```

symbol not starting with special char:

```
abc = (` "abc")
```

Example – List Predicates

Concatenation of sequences:

```
(concat ($1) ($2) ($1 $2)).  
→ (concat (a b) (c d e) (a b c d e))
```

Member of a sequence:

```
(member % ($1 % $2)).  
→ (member b (a b c))
```

Reverse of a sequence:

```
(reverse () ()).  
(reverse (% $seq) ($rev %))<  
(reverse ($seq) ($rev)).  
→ (reverse (u v) (v u))
```

Conclusion

- Specifications – software engineering benefit
 - Smaller, more readable, more reusable, more correct
- Minimal & pure – well-suited to proof
 - Equivalence of specification and program
 - Prove assertions to validate specification
- Billion-dollar application!
 - http://axiomaticlanguage.org/A_Vision_for_CAD_released.pdf
- Implementation grand challenge
 - Transformation of specifications to programs
 - http://axiomaticlanguage.org/LOPSTR18_LM_released.pdf